Christ Church Grammar School

2019
TEST 1

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name_____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(2 marks)**

The curve $y = kx^2 - 7x + 6$ has a gradient of 11 when $x = 3$. Find the value of k.

Question 2 **(3 marks)**

Find the derivative of $\frac{\sin(2\theta)}{\cos(2\theta)}$ with respect to θ. You must show full working with use of the Quotient Rule. Simplify your answer.

See next page

Question 3 **(6 marks)**

Consider the function $f(x) = \frac{1}{\sqrt{x}}$. (2 marks)

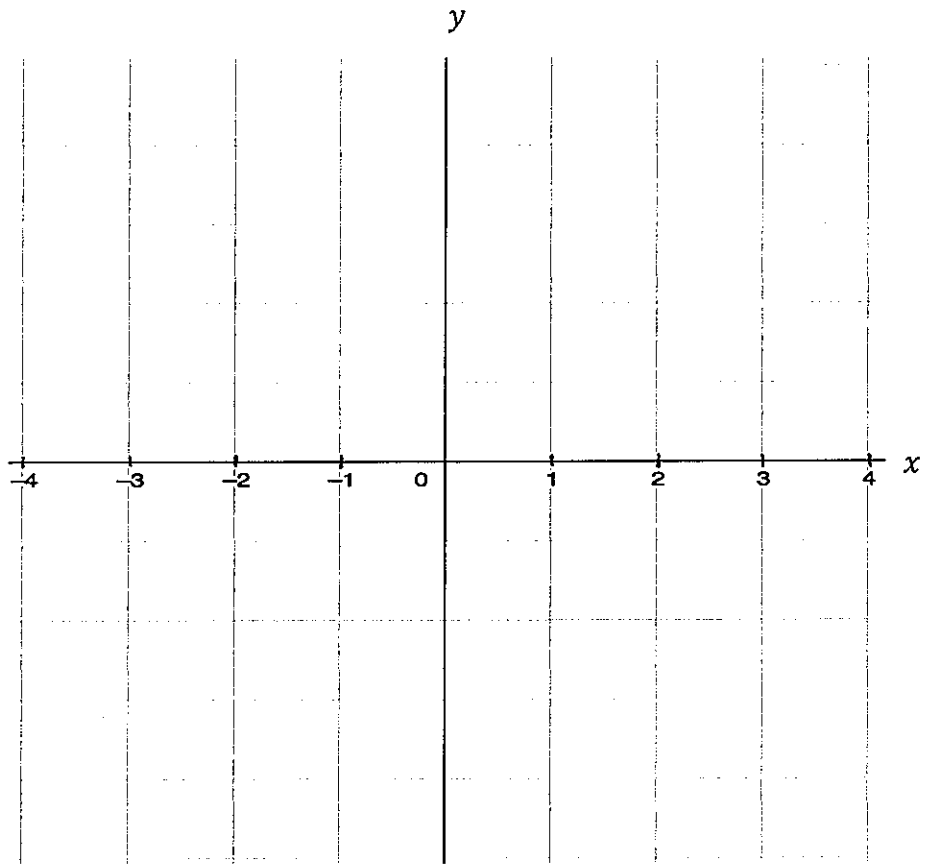
(a) Calculate the derivative of $f(x)$.

(b) Using your answer to part (a) and the function $f(x)$, calculate the approximate value of $\frac{1}{\sqrt{100.5}}$, leaving your answer as a fraction. (4 marks)

Question 4 **(4 marks)**

Sketch a function $y = f(x)$ with all of the following features.

- $f(1) = f(-1) = 0$
- $f'(0) = f'(2) = 0$
- $f''(2) = 0$
- $f'(x) < 0 \ for \ x < 0$
- $f'(x) > 0 \ for \ x > 2, \ 0 < x < 2$



End of Questions

Additional working space

Question number: _____

Additional working space

Question number: _____

See next page

Christ Church Grammar School

2019
TEST 1

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

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Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

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See next page

Question 5 **(8 marks)**

A particle moves in such a way that its displacement x metres from the origin is given by $x = t^3 - 6t^2 + 9t - 1$, where t is the time in seconds.

Determine:

(a) where the particle is initially. (1 mark)

(b) an expression for the velocity of the particle in terms of t. (1 mark)

(c) when the particle is at rest. (2 marks)

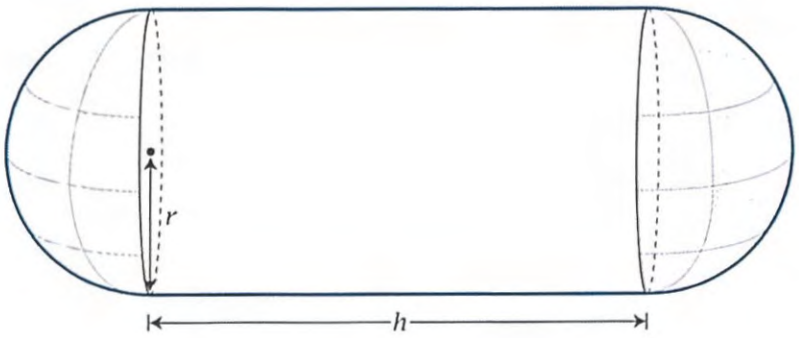
(d) an expression for the acceleration of the particle and the acceleration when $t = 2$ seconds. (2 marks)

(e) the distance travelled in the first 3 seconds. (2 marks)

See next page

Question 6 **(7 marks)**

A tank is to be built with a volume of $200\pi\ m^3$. It is to be built with metal at a cost of $\$p$ per m^2 of cylindrical surface and $\$2p$ per m^2 of hemispherical surface. The tank is to be built according to the diagram shown below:



(a) Show that the cost function $C = \frac{400\pi p}{r} + \frac{16\pi r^2 p}{3}$. (3 marks)

Question 6 continued

(b) Find the radius for the minimum cost of the tank, verifying that it is a minimum. (3 marks)

(c) Find the cost when $p = 10$. (1 mark)

See next page

Question 7 **(5 marks)**

(a) Find $f'(\pi)$, if $f(x) = \frac{3x^2+1}{\cos(x)}$, leaving your answer as an exact value. (2 marks)

(b) A curve with equation $y = ax^3 + bx^2 + cx + d$ has zero gradient at the point $\left(\frac{1}{3}, \frac{4}{27}\right)$ and also touches, but does not cross, the x-axis at the point $(1,0)$. Find the values of x for which the curve has a negative gradient.

(3 marks)

See next page

Question 8 **(5 marks)**

Two variables x and y are such that $x^4y = 8$. A third variable z is defined by $z = x + y$.

(a) Calculate the values of x and y at the point where z is a stationary point.

(3 marks)

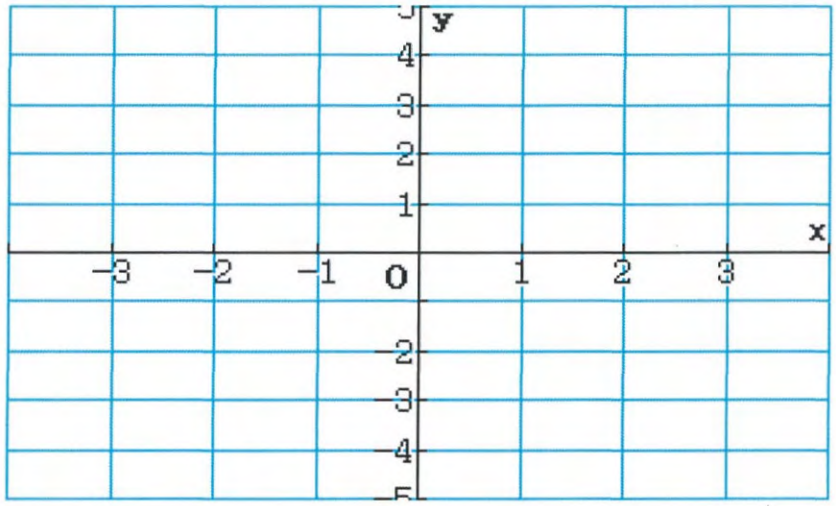
(b) Use the second derivative test to show that this value of z is a minimum.

(2 marks)

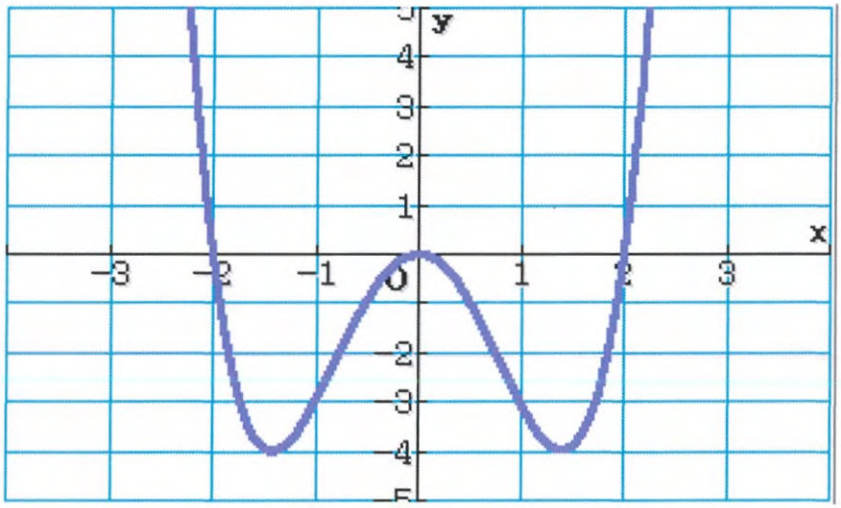
Question 9 **(3, 2 marks)**

Sketch the possible graphs of $f(x)$ and $f''(x)$ on the axes provided below given the graph of the derivative function $f'(x)$.

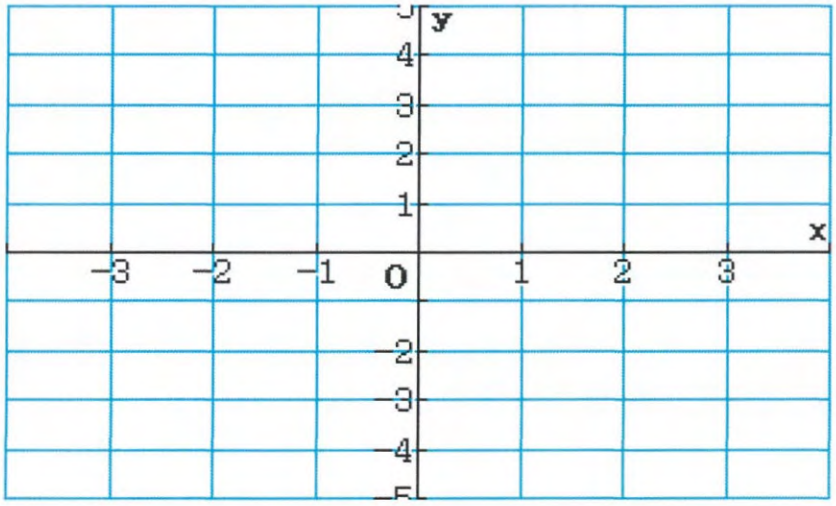
$f(x)$



$f'(x)$



$f''(x)$



End of questions

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Question number: _____

See next page

Additional working space

Question number: _____

See next page

Christ Church Grammar School

2019
UNIT TEST 1

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

FINAL

Your name SOLUTIONS

Teacher's name _____

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7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(2 marks)**

The curve $y = kx^2 - 7x + 6$ has a gradient of 11 when $x = 3$. Find the value of k.

$\frac{dy}{dx} = 2kx - 7$ ✓ $\Rightarrow \left.\frac{dy}{dx}\right|_{x=3} = 11$

$\therefore \; 11 = 2k(3) - 7$

$k = 3$ ✓

Question 2 **(3 marks)**

Find the derivative of $\frac{\sin(2\theta)}{\cos(2\theta)}$ with respect to θ. You must show full working with use of the Quotient Rule. Simplify your answer.

$$\frac{u'v - uv'}{v^2} = \frac{2\cos(2\theta).\cos(2\theta) - \sin(2\theta).(-2\sin(2\theta))}{\cos^2(2\theta)}$$ ✓ Quotient Rule

$$= \frac{2\cos^2(2\theta) + 2\sin^2(2\theta)}{\cos^2(2\theta)}$$

$$= \frac{2\left[\cos^2(2\theta) + \sin^2(2\theta)\right]}{\cos^2(2\theta)}$$ → $\cos^2(2\theta) + \sin^2(2\theta) = 1$ ✓ Pythagorean Identity

$$= \frac{2}{\cos^2(2\theta)}$$ ✓ Simplifies

OR $2\sec^2(2\theta)$

5

Question 4 **(4 marks)**

Sketch a function $y = f(x)$ with all of the following features.

- $f(1) = f(-1) = 0$
- $f'(0) = f'(2) = 0$
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- $f'(x) < 0 \ for \ x < 0$
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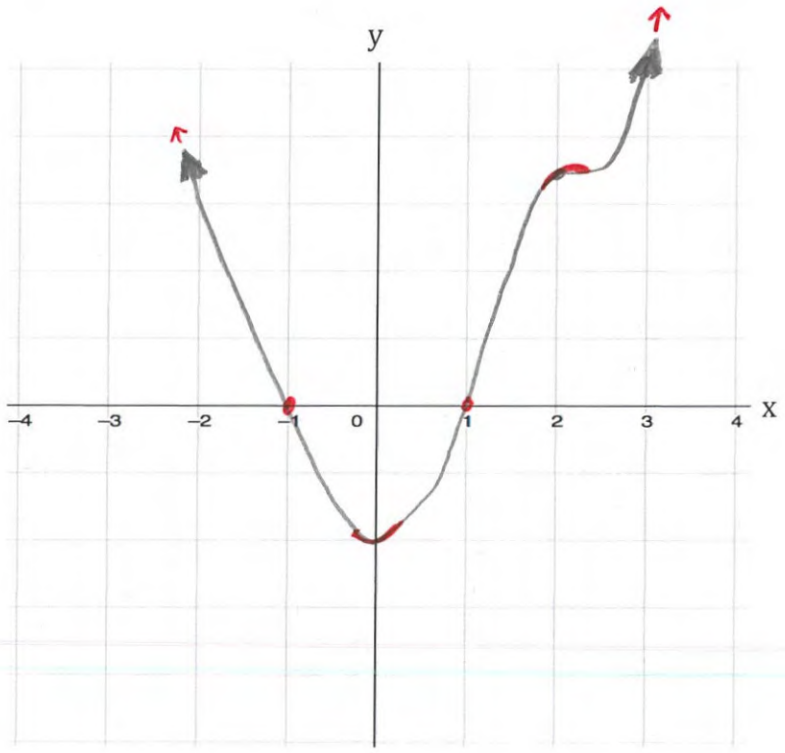

roots ✓

Horiz infl $x=2$ ✓

Min TP ✓

Shape as $x \to \pm\infty$ ✓

4

End of Questions

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UNIT TEST 1

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name - SOLUTIONS -

Teacher's name _____

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Question 5 **(8 marks)**

A particle moves in such a way that its displacement x metres from the origin is given by $x = t^3 - 6t^2 + 9t - 1$, where t is the time in seconds.

Determine:

(a) where the particle is initially. (1 mark)

$x|_{t=0} = -1\ m$ ✓ OR 1 m to left.

(b) an expression for the velocity of the particle in terms of t. (1 mark)

$x'(t) = 3t^2 - 12t + 9 \quad m/s$ ✓

(c) when the particle is at rest. (2 marks)

when $x'(t) = 0$

$0 = 3t^2 - 12t + 9$

$t = 1, 3 \text{ sec}$ ✓✓

(d) an expression for the acceleration of the particle and the acceleration when $t = 2$ seconds. (2 marks)

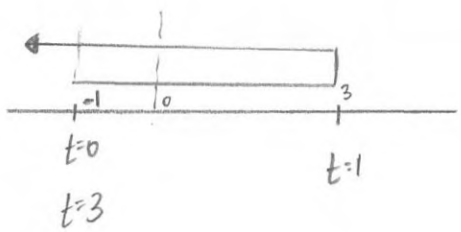
$a = v'(t)$

$a = 6t - 12 \ ms^{-2}$ ✓

$a|_{t=2sec} = 6(2) - 12$

$= 0 m/s^2$ ✓

(e) the distance travelled in the first 3 seconds. (2 marks)



total dist = 8m ✓✓

-1 overall for missing units more than once

See next page

8

Question 6 continued

(b) Find the radius for the minimum cost of the tank, verifying that it is a minimum. (3 marks)

$\frac{dC}{dr} = \frac{32pr^3\pi - 1200\pi p}{3r^2}$ ✓ [Shows $\frac{dC}{dr}$]

$\frac{dC}{dr} = 0 \Rightarrow r = \sqrt[3]{\frac{75}{2}}$

$r = 3.347\text{m}$ ✓ (radius)

Check $\left.\frac{d^2C}{dr^2}\right|_{r=3.347} = 100.54p > 0$ ✓ [2nd deriv. check]

∴ Min

(c) Find the cost when $p = 10$. (1 mark)

$C|_{p=10}$ on CPad = \$5631.50 ✓

4

Question 7 **(5 marks)**

(a) Find $f'(\pi)$, if $f(x) = \frac{3x^2+1}{\cos(x)}$, leaving your answer as an exact value. (2 marks)

$f'(x)\Big|_{x=\pi}$ on Clad $= -6\pi$ ✓✓

(b) A curve with equation $y = ax^3 + bx^2 + cx + d$ has zero gradient at the point $\left(\frac{1}{3}, \frac{4}{27}\right)$ and also touches, but does not cross, the x-axis at the point $(1,0)$. Find the values of x for which the curve has a negative gradient.

(3 marks)

$y' = 3ax^2 + 2bx + c$

Sub y' $(\frac{1}{3}, 0)$ $\Rightarrow$ 1. $0 = 3a\left(\frac{1}{3}\right)^2 + 2b\left(\frac{1}{3}\right) + c$

Sub y $(1, 0)$ $\Rightarrow$ 2. $0 = a + b + c + d$

Sub y' $(1, 0)$ $\Rightarrow$ 3. $0 = 3a + 2b + c$

Sub y $(\frac{1}{3}, \frac{4}{27})$ $\Rightarrow$ 4. $\frac{4}{27} = \left(\frac{1}{3}\right)^3 a + \left(\frac{1}{3}\right)^2 b + \left(\frac{1}{3}\right) c + d$

Solve on Clad ✓ [Sets up equations]

$a = 1,\ b = -2,\ c = 1,\ d = 0$ ✓ [Solves for variables]

$\therefore y = x^3 - 2x^2 + x$

Graph on Clad

$\therefore$ Neg. Gradient for $\{x : \frac{1}{3} < x < 1\}$ ✓

5

See next page

Question 8 **(5 marks)**

Two variables x and y are such that $x^4y = 8$. A third variable z is defined by $z = x + y$.

(a) Find the values of x and y that give z a stationary value. (3 marks)

$x^4y = 8 \Rightarrow y = \frac{8}{x^4}$

$\therefore z = x + 8x^{-4}$ ✓ (z in terms of 1 variable)

$\frac{dz}{dx} = 1 - \frac{32}{x^5}$ for stat pt $\frac{dz}{dx} = 0$

$\therefore \frac{32}{x^5} = 1$

$x = 2$, hence $y = \frac{1}{2}$

✓ (x-value) ✓ (y-value)

(b) Use the second derivative test to show that this value of z is a minimum. (2 marks)

$\frac{d^2z}{dx^2} = \frac{160}{x^6}$ ✓ (shows $\frac{d^2z}{dx^2}$)

$\left.\frac{d^2z}{dx^2}\right|_{x=2} = \frac{160}{2^6} > 0$ $\therefore$ Min

✓ ($\frac{d^2z}{dx^2} > 0$)

5

Question 3 **(6 marks)**

Consider the function $f(x) = \frac{1}{\sqrt{x}}$. (2 marks)

(a) Calculate the derivative of $f(x)$.

$f(x) = x^{-1/2}$

$f'(x) = -\frac{1}{2}x^{-3/2}$ ✓✓ (correct derivative)

OR $f'(x) = -\frac{1}{2\sqrt{x^3}}$

(b) Using your answer to (a) and the function $f(x)$, calculate the approximate value of $\frac{1}{\sqrt{100.5}}$, leaving your answer as a fraction. (4 marks)

When $x = 100 \Rightarrow y = \frac{1}{10}$ & $\frac{dy}{dx} = -\frac{1}{2}(100)^{-3/2}$

$\frac{dy}{dx} = -\frac{1}{2000}$ ✓ $\left.\frac{dy}{dx}\right|_{x=1000}$

Using $\frac{dy}{dx} \approx \frac{\delta y}{\delta x} \Rightarrow \delta y \approx \frac{dy}{dx} \times \delta x$

$\delta y \approx -\frac{1}{2000} \times 0.5$ ✓ [Incremental Formula]

for $x = 100$
$\delta x = 0.5$

$\therefore \delta y \approx -\frac{1}{4000}$ ✓ (δy)

Hence $\frac{1}{\sqrt{100.5}} = \frac{1}{10} - \frac{1}{4000}$

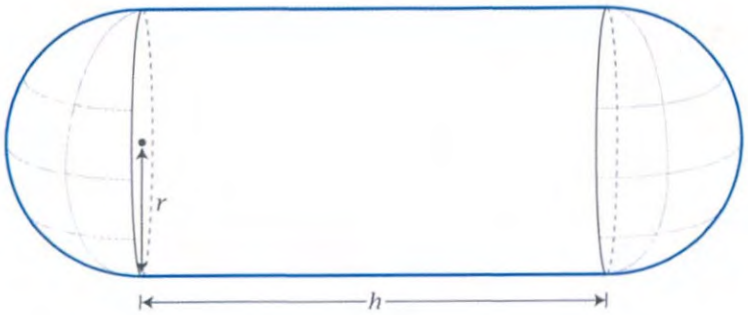
$= \frac{399}{4000}$ ✓ (Fraction)

6

See next page

Question 6 **(7 marks)**

A tank is to be built with a volume of $200\pi\ m^3$. It is to be built with metal at a cost of $\$p$ per m^2 of cylindrical surface and $\$2p$ per m^2 of hemispherical surface. The tank is to be built according to the diagram shown below:



(a) Show that the cost function $C = \frac{400\pi p}{r} + \frac{16\pi r^2 p}{3}$. (3 marks)

$V = 200\pi$

$\therefore 200\pi = \frac{4}{3}\pi r^3 + \pi r^2 h \longrightarrow h = \frac{200\pi - \frac{4}{3}\pi r^3}{\pi r^2}$

✓ (isolates h from volume)

$h = \frac{200}{r^2} - \frac{4}{3}r$

Cost $SA = 2\pi rh(p) + 4\pi r^2(2p)$

sub in h. ✓ (SA expression)

$SA = 2\pi rp\left(\frac{200}{r^2} - \frac{4}{3}r\right) + 8\pi r^2 p$ (h)

$= \frac{400\pi rp}{r^4} - \frac{8\pi r^2 p}{3} + \frac{24}{3}\pi r^2 p$ ✓ (Subs in h and simplifies)

$= \frac{400\pi p}{r} + \frac{16\pi r^2 p}{3}$

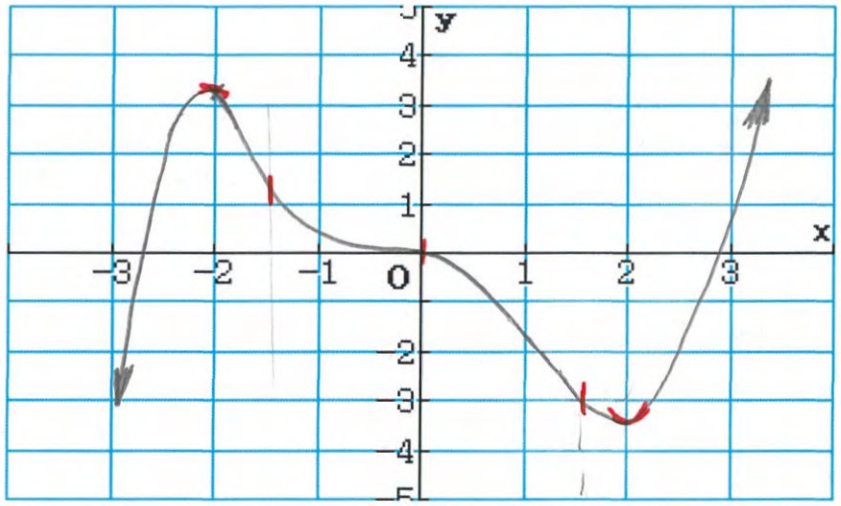
As required.

3

Question 9 **(5 marks)**

Sketch the possible graphs of $f(x)$ and $f''(x)$ on the axes provided below given the graph of the derivative function $f'(x)$.

(a) $f(x)$



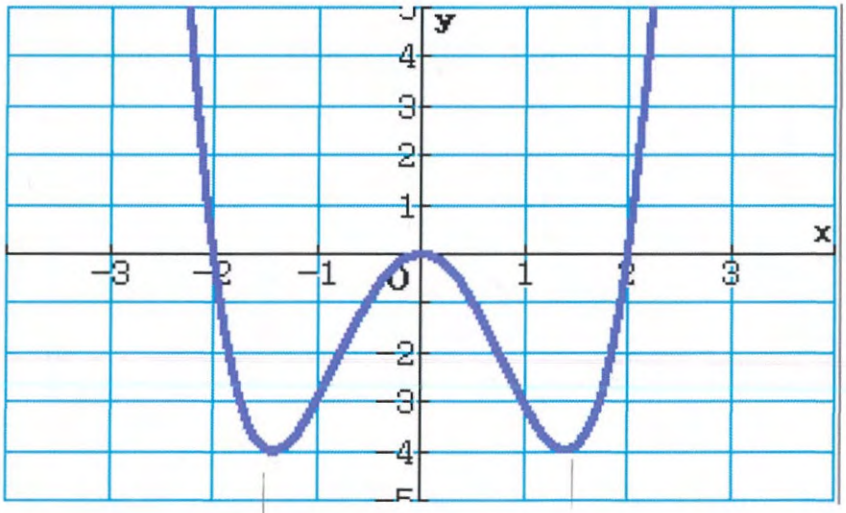
(3 marks)

(Max / Min @ $x = -2$, $x = 2$) ✓

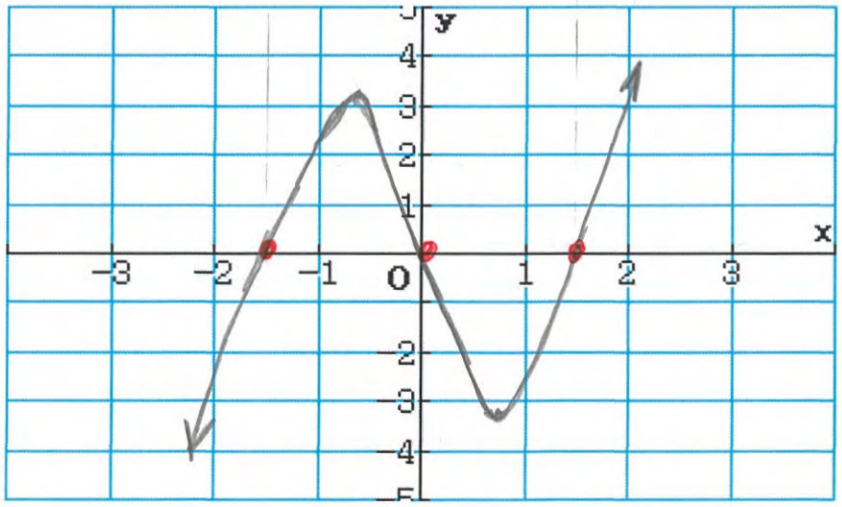
(Inflection pts in line with f'(x) min) ✓

(General Shape) ✓

$f'(x)$



(b) $f''(x)$



(2 marks)

Roots ✓

Shape ✓

End of questions

5